# ADAS Control Algorithm Improvements

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## 1. Introduction

Will include the assumptions, variable definitions, algorithm structure We are assuming constant air pressure, gravitational force and that the rocket is flying straight up. The drag force is proportional to $v^2$, ie the coefficient of drag is independent of velocity, although this could be accounted for by using a 2 dimensional lookup table rahter than a 1 dimensional one.

## 2. Maths

At any point after burnout, so during the coasting phase, we wish to calculate the deployment of ADAS needed to reach the desired altitude at apogee. Note that this depolyment could be fed directly into the motor or could go through a PID algorithm which will account for offsets and such (especially the P and I components of it at least). We do this by considering the forces acting on the rocket and applying Newton's Second Law.

The forces acting on the rocket are the gravitational force and the drag force.

$$m\left(\frac{dx^2}{dt^2}\right) = -mg - \frac{1}{2}\rho C_d A\left(\frac{dx}{dt}\right)^2$$

$$\frac{dx^2}{dt^2} = -g - \frac{\rho}{2m}\left(C_d A\right)\left(\frac{dx}{dt}\right)^2$$

Now define: $\alpha = \frac{\rho}{2m}$ this is a constant $\Delta = C_d A$ this depends on the deployment of the ADAS fins.

So this problem equates to solving for $\Delta$ given the initial conditions and the target. To solve this DE first convert it to a first order equation in terms of $v = \frac{dx}{dt}$

$$\frac{dv}{dt} = -g - \alpha\Delta v^2$$

$$\frac{1}{g + \alpha\Delta v^2}\frac{dv}{dt} = -1$$

$$\frac{1}{1 + \frac{\alpha\Delta}{g}v^2}\frac{dv}{dt} = -\frac{1}{g}$$

Rescale the velocity variable to be able to integrate, define:

$$\nu = \sqrt{\frac{\alpha\Delta}{g}}v \qquad \frac{dv}{dt} = \sqrt{\frac{g}{\alpha\Delta}}\frac{d\nu}{dt}$$

And so we have that:

$$\frac{1}{1+\nu^2}\sqrt{\frac{g}{\alpha\Delta}}\frac{d\nu}{dt} = -\frac{1}{g}$$

$$\frac{1}{1+\nu^2}\frac{d\nu}{dt} = -\sqrt{\frac{\alpha\Delta}{g^3}}$$

Now integrating both sides

$$\int \frac{1}{1+\nu^2}\frac{d\nu}{dt}dt = \int -\sqrt{\frac{\alpha\Delta}{g^3}}dt$$

$$tan^{-1}(\nu) = -\sqrt{\frac{\alpha\Delta}{g^3}}t + c$$

Now substituting in the initial condition at $t = 0$ whish is the current time we have $v = v_0$ and so using the above equation $\nu = \nu_0$

$$c = tan^{-1}(\nu_0)$$

Now resubstituting in $\nu$ for its definition in terms of x

$$\frac{dx}{dt} = \sqrt{\frac{g}{\alpha\Delta}} tan\left(-\sqrt{\frac{\alpha\Delta}{g^3}} t + c\right)$$

From here we simply integrate with respect to t to get x as a function of t

$$x = \int \sqrt{\frac{g}{\alpha\Delta}} tan\left(-\sqrt{\frac{\alpha\Delta}{g^3}} t + c\right) dt$$

Since $tan(y) \equiv \frac{sin(y)}{cos(y)}$ and making a trig substitution we can get the following:

$$x = \sqrt{\frac{g}{\alpha\Delta}} \left(-\frac{ln\left(cos\left(-\sqrt{\frac{\alpha\Delta}{g^3}} t + c\right)\right)}{-\sqrt{\frac{\alpha\Delta}{g^3}}}\right) + X$$

Using the initial condition that $x(0) = x_0$ which is the height measured by the sensors at that instance we get

$$x_0 = \sqrt{\frac{g}{\alpha\Delta}} \left(\frac{ln\left(cos\left(c\right)\right)}{\sqrt{\frac{\alpha\Delta}{g^3}}}\right) + X$$

and hence

$$X = x_0 - \frac{g^2}{\alpha\Delta} ln\left(cos\left(c\right)\right)$$

We are interested in the height reached at apogee. Apogee occurs when $\frac{dx}{dt} = v = 0$. Solving this equation from the expression given above:

$$\frac{dx}{dt} = \sqrt{\frac{g}{\alpha\Delta}} tan\left(-\sqrt{\frac{\alpha\Delta}{g^3}} t_t + c\right) = 0$$

$$-\sqrt{\frac{\alpha\Delta}{g^3}} t_t + c = tan^{-1}(0) = 0$$

Only taking the first solution

$$t_t = c\sqrt{\frac{g^3}{\alpha\Delta}}$$

Now at the top, $t = t_t$ and $x = x_t$ where $x_t$ is the target height. Plugging these into the equation for height

$$x_t = \frac{g^2}{\alpha\Delta} ln\left(cos\left(-\sqrt{\frac{\alpha\Delta}{g^3}} t_t + c\right)\right) + X$$

$$x_t = \frac{g^2}{\alpha\Delta} ln\left(cos\left(0\right)\right) + X$$

$$x_t = \frac{g^2}{\alpha\Delta} ln\left(1\right) + X$$

$$x_t = \frac{g^2}{\alpha\Delta} \left(\frac{ln\left(1\right)}{\sqrt{\frac{\alpha\Delta}{g^3}}}\right) + X$$

$$x_t = X = x_0 - \frac{g^2}{\alpha\Delta} ln\left(cos\left(c\right)\right)$$

$$x_0 - x_t = \frac{g^2}{\alpha\Delta} ln\left(cos\left(c\right)\right)$$

$$\Delta = \frac{g^2}{\alpha\left(x_0 - x_t\right)} ln\left(cos\left(c\right)\right)$$

Since $c$ depends on $\Delta$ in a non-trivial way this is a transcendental equation for $\Delta$ which should be solved numerically? to get the appropriate deployment in order to reach apogee (future)

Since $tan(c) = \nu_0$ then $cos(c) = \frac{1}{\sqrt{1+\nu_0^2}}$ and so:

$$\Delta = \frac{g^2}{\alpha\left(x_0 - x_t\right)} ln\left(\frac{1}{\sqrt{1 + \nu_0^2}}\right)$$

$$\Delta = \frac{g^2}{\alpha\left(x_0 - x_t\right)} ln\left(\frac{1}{\sqrt{1 + \frac{\alpha\Delta}{g} v_0^2}}\right)$$

$$\Delta - \frac{g^2}{\alpha\left(x_0 - x_t\right)} ln\left(\frac{1}{\sqrt{1 + \frac{\alpha\Delta}{g} v_0^2}}\right) = 0$$

Newton's method can be used to calculate the roots of this equation. Need to show that newton's method will converge to the root for good enough guesses with constraints of when it will converge and make sure the algorithm only initially guesses a good enough value

$$\Delta \frac{\alpha\left(x_0 - x_t\right)}{g^2} = ln\left(\frac{1}{\sqrt{1 + \frac{\alpha\Delta}{g} v_0^2}}\right)$$

$$e^{\Delta \frac{\alpha(x_t - x_0)}{g^2}} = \sqrt{1 + \frac{\alpha v_0^2}{g}\Delta}$$

$$e^{\Delta \frac{\alpha(x_t - x_0)}{g^2}} - \sqrt{1 + \frac{\alpha v_0^2}{g}\Delta} = 0$$

Taking this as the root of a function

$$f(\Delta) = e^{\Delta \frac{\alpha(x_t - x_0)}{g^2}} - \sqrt{1 + \frac{\alpha v_0^2}{g}\Delta} = 0$$

$$f'(\Delta) = \frac{\alpha\left(x_t - x_0\right)}{g^2} e^{\Delta \frac{\alpha(x_t - x_0)}{g^2}} - \frac{1}{2}\frac{\frac{\alpha v_0^2}{g}}{\sqrt{1 + \frac{\alpha v_0^2}{g}\Delta}}$$

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$$f'(\Delta) = \frac{\alpha\left(x_t - x_0\right)}{g^2} e^{\Delta \frac{\alpha(x_t - x_0)}{g^2}} - \frac{\alpha v_0^2}{2g\sqrt{1 + \frac{\alpha v_0^2}{g}\Delta}}$$

Now this equation can be solved numerically using Newton's Method, guessing and applying a first order taylor expansion to get the next guess